PENNSYLVANIA MATHEMATICS LEAGUE

P.O. Box 7136, Elkins Park, Pennsylvania 19027-0136

All official participants must take this contest at the same time.

Contest Number 1 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **October 19, 2010**

Name ______________________ **Teacher** ___________ **Grade Level** _____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: NOV. 16, 2010 *Answer Column*

1-1. The sum of the three smallest primes and one other prime is 77. What is the product of these four primes?

1-1.

1-2. What is the smallest perfect square greater than 1 million?

1-2.

1-3. What are both integers n for which
$$(2^{n^4})(2^{n^3})(2^{n^2})(2^{n}) = 1?$$

1-3.

1-4. The cost (in cents) of 8 candies is equal to the number of candies that I can buy for 98 cents. At the same cost per candy, how many cents do 14 candies cost?

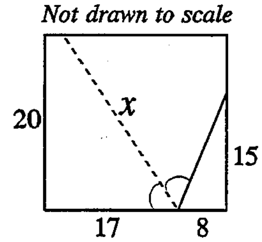
1-4.

1-5. Of the integers between 10^3 and 10^4 that have no repeated digit, how many have digits that increase from left to right?

1-5.

1-6. In the rectangle at the right, the dotted line segment bisects the obtuse angle through which it is drawn, as shown. What is the length of this dotted line segment, if the other line segments have lengths as marked?

Not drawn to scale



1-6.

PENNSYLVANIA MATHEMATICS LEAGUE

P.O. Box 7136, Elkins Park, Pennsylvania 19027-0136

All official participants must take this contest at the same time.

Contest Number 2 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **November 16, 2010**

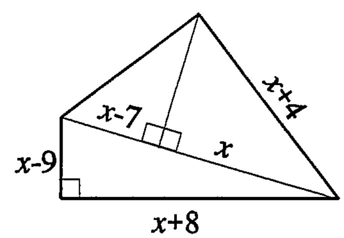
Name ______________________ **Teacher** ___________ **Grade Level** _____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: DEC. 14, 2010 *Answer Column*

2-1. If $x^2 + 6x + 5 = 0$, what is the value of $10x^2 + 60x$? 2-1.

2-2. For what value of $k > 0$ will the triangle with vertices at (0,0), $(k,0)$, and (2010,2010) have an area of 2010^2? 2-2.

2-3. In the quadrilateral shown, one of the diagonals is drawn, some of the line segments are marked with their lengths, and some line segments are marked as being perpendicular. What is the perimeter of the quadrilateral? 2-3.



2-4. A bag of 5 apples, 7 bananas, and 3 carrots costs \$4.41; and a bag of 6 apples, 2 bananas, and 1 carrot costs \$2.37. At these same prices, how much should a bag of 3 apples, 17 bananas, and 7 carrots cost? 2-4.

2-5. Factor x^4+4; that is, write x^4+4 as a product of two quadratic polynomials with integral coefficients. 2-5.

2-6. What are all pairs of positive integers (a,b) for which a^2+b exceeds $a+b^2$ by 36? 2-6.

PENNSYLVANIA MATHEMATICS LEAGUE

P.O. Box 7136, Elkins Park, Pennsylvania 19027-0136

All official participants must take this contest at the same time.

Contest Number 3

Any calculator without a QWERTY keyboard is allowed. Answers must be exact *or* have 4 (or more) significant digits, correctly rounded.

December 14, 2010

Name ______________________ Teacher ___________ Grade Level _____ Score ____

Time Limit: 30 minutes NEXT CONTEST: JAN. 11, 2011 *Answer Column*

3-1. Today, you and I each independently picked a whole number from 1 through 9. Whether we divide my pick by yours or your pick by mine, we get the same remainder. What is this remainder?

3-1.

3-2. What is the least possible perimeter of a triangle whose side-lengths are consecutive perfect squares?

3-2.

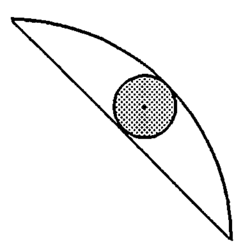
3-3. What is the numerical measure of a parallelogram's fourth angle if its other three angles have degree-measures $2x+30$, $3x+50$, and $4x-10$?

3-3.

3-4. What is the smallest positive number y for which $(\sqrt{2})(\sqrt{4})(y)$ and $(\sqrt{3})(\sqrt{6})(y)$ both have positive integral values?

3-4.

3-5. What is the area of the largest circle that can be drawn as shown, tangent to both a quarter-circle (a 90° arc) of length 2π and the line segment that connects the endpoints of that quarter-circle?



3-5.

3-6. At random, I choose 3 different points from among 2010 points evenly spaced on a circle. What is the probability that the 3 points I choose are the vertices of a right triangle?

3-6.

PENNSYLVANIA MATHEMATICS LEAGUE

P.O. Box 7136, Elkins Park, Pennsylvania 19027-0136

All official participants must take this contest at the same time.

Contest Number 4 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **January 11, 2011**

Name ____________________ **Teacher** __________ **Grade Level** _____ **Score** ____

Time Limit: 30 minutes NEXT CONTEST: FEB. 22, 2011 *Answer Column*

4-1. What is the (numerical) area of a square in which two sides have lengths $x-4$ and $x^2-7x+11$?

4-1.

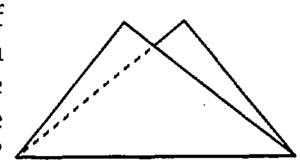
4-2. At least how many dogs do I own if more than half the dogs I own are male, and more than 40% of the dogs I own are female?

4-2.

4-3. From the 99 positive integers less than 100, I chose as many different numbers as I could so that no subset of my numbers had a sum of 100. If the sum of all my numbers was as large as possible, what was the smallest number I actually chose?

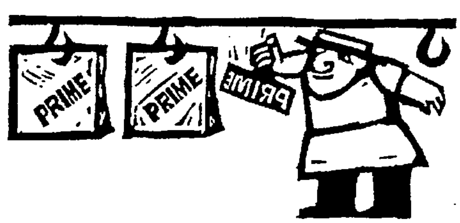
4-3.

4-4. A sheet of 8×10 paper is folded along one of its diagonals, with the crease running from corner to corner as shown. What is the length of the dotted line (that part of the longer side hidden from view after the fold)?



4-4.

4-5. For what prime p is $2003p+16$ the square of an integer?



4-5.

4-6. What is the only polynomial f with integral coefficients that satisfies $f(\sqrt{3}+\sqrt{2}) = \sqrt{3}-\sqrt{2}$ and is of least degree ≥ 1?

4-6.

PENNSYLVANIA MATHEMATICS LEAGUE

P.O. Box 7136, Elkins Park, Pennsylvania 19027-0136

All official participants must take this contest at the same time.

Contest Number 5 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **February 22, 2011**

Name ____________________ **Teacher** __________ **Grade Level** ____ **Score** ___

Time Limit: 30 minutes NEXT CONTEST: MAR. 22, 2011 *Answer Column*

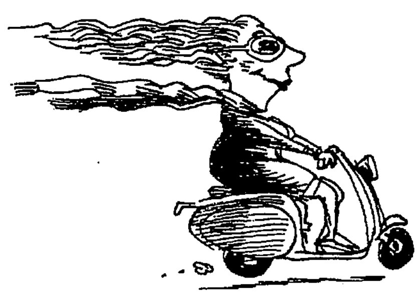
5-1. How long is a side of an equilateral triangle whose area equals the sum of the areas of two equilateral triangles with sides 3 and 4?

5-1.

5-2. If m and n are different positive integers, and if $\dfrac{\frac{1}{m}-\frac{1}{n}}{1-\frac{1}{mn}}=1$, what is the value of m?

5-2.

5-3. Four girls raced scooters. Alice said, "I was first." Barb said, "I was not last." Cathy said, "I was last." Di said, "I was neither first nor last." If three girls told the truth and one girl lied, who came in first?

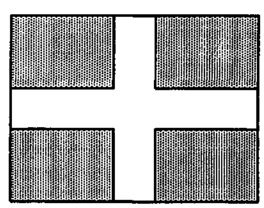


5-3.

5-4. A 2011-sided regular polygon has both its inscribed and circumscribed circles drawn. If the length of each side of the polygon is 2, what is the area of the region between the two circles?

5-4.

5-5. In my 6×8 rectangular garden, the paths, shown unshaded, have equal widths. My garden's planting regions are shown as shaded rectangles. If the total areas of the shaded and unshaded regions are equal, how wide is each garden path?



5-5.

5-6. What are all values of k for which at least one pair of real numbers (x,y) satisfies $\sin x + \cos y - \sin x \cos y = k$?

5-6.

PENNSYLVANIA MATHEMATICS LEAGUE

P.O. Box 7136, Elkins Park, Pennsylvania 19027-0136

All official participants must take this contest at the same time.

Contest Number 6 *Any calculator without a QWERTY keyboard is allowed.* Answers must be exact *or* have 4 (or more) significant digits, correctly rounded. **March 22, 2011**

Name ______________________ **Teacher** ___________ **Grade Level** _____ **Score** ____

Time Limit: 30 minutes FINAL CONTEST OF THE YEAR *Answer Column*

6-1. The least common multiple of the first 5 positive integers is 60. What is the least common multiple of the first 6 positive integers? 6-1.

6-2. What is the integer r for which $x^3-49x^2+140x-92=(x-1)(x-2)(x-r)$ for all values of x? 6-2.

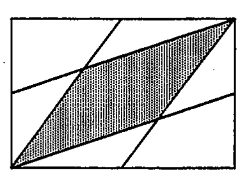
6-3. What is the greatest positive integer that can be the value of x in a triangle whose sides have lengths $\log 4$, $\log 503$, and $\log x$? 6-3.

6-4. The numbers on Al's, Bo's, and Cy's uniforms add up to 21. Cy's number (the largest) exceeds Bo's number by as much as Bo's exceeds Al's. If Cy's were 1 more and Bo's were 1 less, then the ratio of Cy's new number to Bo's new number would be the same as the ratio of Bo's new number to Al's number. What is Al's number? 6-4.

6-5. Although your class has more kids than mine, we can both make the statement below. Altogether how many kids are in our two classes? 6-5.

In my class, each boy has as many dollars as the number of boys; and each girl has as many dollars as the number of girls. Altogether, the kids in my class have a total of \$697.

6-6. Connect each of two opposite vertices of a rectangle to the midpoints of both sides not containing that vertex, as shown. If the area of the rectangle is 360, what is the area of the region thus determined (shown shaded)? 6-6.



 Visit our Web site at www.mathleague.com

Contest # 1 — *Answers & Solutions* — 10/19/10

Problem 1-1

Three of the primes are the smallest possible primes. Since 1 is not a prime, three of the four primes must be 2, 3, and 5. The fourth prime is $77-5-3-2=67$. The product of all four is $2\times3\times5\times67=\boxed{2010}$.

Problem 1-2

The square root of 1 million is 1 thousand. Add 1 and square it to get $\boxed{(1001)^2 \text{ or } 1\,002\,001}$.

Problem 1-3

If $(2^{n^4})(2^{n^3})(2^{n^2})(2^{n})=1=2^0$, the sum of the exponents is $n^4+n^3+n^2+n=n(n^3+n^2+n+1)=n(n+1)(n^2+1)=0$. Since n is real, $n=\boxed{0,\ -1}$.

Problem 1-4

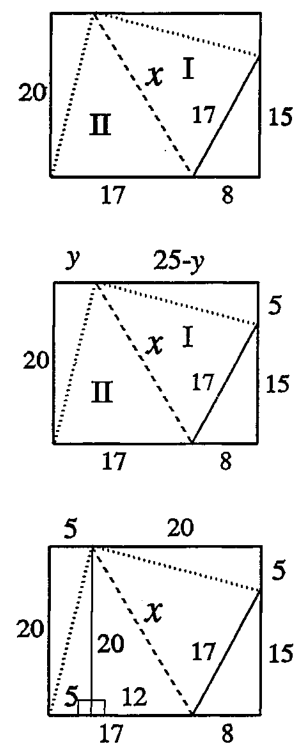
Method I: Let c be the cost of one candy, in cents. Then, $8c=\frac{98}{c}$, so $c^2=\frac{98}{8}=\frac{49}{4}$. Thus, $c=\frac{7}{2}$, and $14c=\boxed{49 \text{ or } 49¢}$.

Method II: If n candies can be bought for 98¢, then the number of cents that each candy costs is $\frac{98}{n}$. But n is also the number of cents that it costs to buy 8 candies, so the cost of each, in cents, is $\frac{n}{8}$. Equating, $\frac{98}{n}=\frac{n}{8}$, so $n=\sqrt{2\cdot4\cdot2\cdot49}=2\cdot2\cdot7=28$. Since 28 is the number of candies that can be bought for 98¢, the cost of 14 candies would be 49¢.

Problem 1-5

For each selection of 4 different digits, there's only one way to arrange them in increasing order, left to right. So the question becomes: How many ways can we select the 4 digits? Since the leftmost digit is a 1 or a larger digit, the chosen digits cannot include a 0. Therefore, we must choose 4 of the 9 digits other than 0. Hence, the total number of integers between 1 thousand and 10 thousand whose digits increase from left to right is $\binom{9}{4}=\boxed{126}$.

Problem 1-6



In the scale diagrams, $\triangle$I and $\triangle$II share a common side that bisects the obtuse angle through which the side is drawn. That makes $\triangle\text{I}\cong\triangle\text{II}$ by SAS, so the two dotted lines we added to the diagram are congruent. Next, by the Pythagorean Theorem, $20^2+y^2=(25-y)^2+5^2$, from which $y=5$. In the third diagram, we can use the Pythagorean Theorem to get $x^2=12^2+20^2=544$, so $x=\boxed{\sqrt{544}}=4\sqrt{34}$.

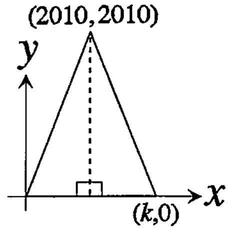
Contest # 2 *Answers & Solutions* 11/16/10

Problem 2-1

Since $x^2+6x = -5$, $10x^2+60x = -5\times 10 = \boxed{-50}$.

Problem 2-2

A quick sketch of the triangle on a pair of rectangular coordinate axes would look like the sketch at the right (which is not drawn to scale). Draw an altitude from (2010,2010) to the x-axis. The length of that altitude is 2010.

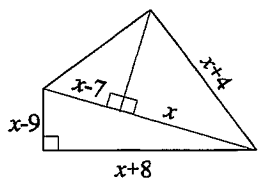


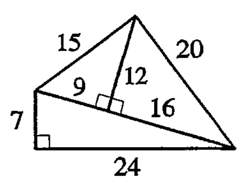
Thus, the area of the triangle $= 2010k/2 = 2010^2$. That will happen when $k = \boxed{4020}$.

[NOTE: If $k < 0$ were permitted, then another solution would be $k = -4020$.]

Problem 2-3

By the Pythagorean Theorem, $(x-9)^2 + (x+8)^2 = (x-7+x)^2$, so $x = 16$ and we get the next diagram.





The perimeter of the quadrilateral is $7 + 15 + 20 + 24 = \boxed{66}$.

Problem 2-4

Let's see how we might get the required 3 apples and other fruit from pre-filled bags of 5 apples and other fruit or 6 apples and other fruit. One way is to buy 3 five-apple bags and sell 2 six-apple bags to my friend. In fact, $3(5a+7b+3c = \$4.41) - 2(6a+2b+c = \$2.37)$ gives us the result $3a+17b+7c = \boxed{\$8.49}$.

[NOTE: This problem would have no solution unless the contents and price of the new collection of fruit was dependent on the two conditions in the first sentence of the problem. The dependency can be seen in the linear combination $x(5a+7b+3c = \$4.41) + y(6a+2b+c = \$2.37)$, from which $a(5x+6y)+b(7x+2y)+c(3x+y) = \$4.41x+\$2.37y$. We want the value of $3a+17b+7c$, so we want $5x+6y = 3$, $7x+2y = 17$, and $3x+y = 7$. This system is consistent when $x = 3$ and $y = -2$. Finally $\$4.41x + \$2.37y = \$13.23 - \$4.74 = \$8.49$.]

Problem 2-5

$$\begin{aligned} x^4+4 &= (x^4+4x^2+4)-(4x^2) \\ &= (x^2+2)^2 - (2x)^2 \\ &= (x^2+2+2x)(x^2+2-2x) \\ &= \boxed{(x^2+2x+2)(x^2-2x+2)}. \end{aligned}$$

[NOTE: Alternatively, find the fourth roots of -4, then multiply pairs of linear factors with conjugate roots.]

Problem 2-6

$$\begin{aligned} &(a^2+b)-(a+b^2) = 36 \Leftrightarrow \\ &(a^2-b^2)-(a-b) = 36 \Leftrightarrow \\ &(a+b)(a-b)-(a-b) = 36 \Leftrightarrow \\ &(a-b)(a+b-1) = 36. \end{aligned}$$

Since a and b are positive integers, the larger factor, $(a+b-1)$, is a positive integer. Thus, the factor $(a-b)$ is also positive. One of these factors must be even; the other must be odd. Factoring numerically, we get:

$a+b-1 =$	36	12	9	(one factor's values)
$a-b =$	1	3	4	(the other factor's values)

So, $2a-1 = 37\ \ 15\ \ 13$,
$a = 19\ \ 8\ \ 7$

Finally, $(a,b) = \boxed{(19,18),\ (8,5),\ (7,3)}$.

Problem 3-1

If we didn't pick the same numbers, then one of us chose a number larger than the other's. When we divide the smaller by the larger, the remainder is the smaller. But if you divide the larger by the smaller, the remainder would have to be smaller than the smaller. OOPS. What happened? Well, we began by supposing we didn't pick the same numbers. That supposition must have been false, so we must have picked the same numbers. The remainder is $\boxed{0}$.

Problem 3-2

We need three consecutive squares in which the largest is less than the sum of the other two. Here's a list: 1 4 9 16 25 36 49. The smallest three that could be sides of a triangle are 16, 25, and 36. The perimeter of that triangle is $\boxed{77}$.

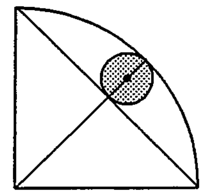
Problem 3-3

Two of the angles must be congruent since the figure is a parallelogram. If $2x+30 = 3x+50$, then $x = -20$, which yields impossible angles. If $2x+30 = 4x-10$, then $x = 20$. If $3x+50 = 4x-10$, then $x = 60$, which is not possible, because the resulting angles are not possible. The three given angles have measures 70, 110, and 70. The degree-measure of the fourth angle is $\boxed{110}$.

Problem 3-4

Let $2y\sqrt{2} = a$ and let $3y\sqrt{2} = b$, where a and b are positive integers. Clearly, $3a = 2b$, so the least possible values of a and b are 2 and 3 respectively. Substitute these into the equations in the first sentence to get $y = \boxed{1/\sqrt{2}}$.

Problem 3-5



The largest such circle is the circle (of radius r) shaded at the right. The quarter-circle (radius R) has arc-length 2π, so $\frac{2\pi R}{4} = 2\pi$ and $R = 4$. By the Pythagorean Theorem, the segment connecting the endpoints of the quarter-circle has length $4\sqrt{2}$. In the big right triangle, the median to the hypotenuse is an altitude, so its length is half the hypotenuse $= 2\sqrt{2}$. The shaded circle's diameter has length $4-2\sqrt{2}$, so $r = (4-2\sqrt{2})/2$. Finally, the area of the shaded circle shown is $\pi r^2 = \boxed{\pi(6-4\sqrt{2}) \text{ or } 1.07802416891\ldots}$.

Problem 3-6

Method I: The points will be vertices of a right triangle if and only if two of them are diametrically opposite. There are 2 cases; we'll add their probabilities together. Pick any point. Of the remaining 2009 points, one is diametrically opposite the first. If it is chosen, then the third point may be any other point. That has probability $1\times\frac{1}{2009}\times 1 = \frac{1}{2009}$. If the second point is not diametrially opposite the first point ($p = 1 - \frac{1}{2009} = \frac{2008}{2009}$), then the third point could match the first ($p = \frac{1}{2008}$) or second ($p = \frac{1}{2008}$), with probability $1\times\left(\frac{2008}{2009}\right)\times\left(\frac{2}{2008}\right) = \frac{2}{2009}$. Now, add $\frac{1}{2009}$ from before. Their sum is $\boxed{\frac{3}{2009}}$.

Method II: There are 1005 pairs of diametrically opposite points. We want to select any pair and any other point. The number of ways this can be done is $\binom{1005}{1}\binom{2008}{1}$. The total number of ways one can select 3 points from among 2010 is $\binom{2010}{3}$. The quotient is $\frac{3}{2009}$.

Contest # 4 *Answers & Solutions* 1/11/11

Problem 4-1

The sides of a square are congruent, so $x^2-7x+11 = x-4 \Leftrightarrow x^2-8x+15 = (x\text{-}3)(x-5) = 0$. When $x = 3$, the sides have negative lengths; so $x = 5$, the length of each side is 1, and the square's area is $\boxed{1}$.

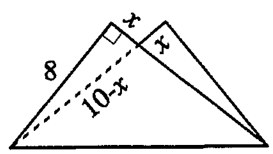
Problem 4-2

The fraction of female dogs is greater than 40% and less than 50%, so its value is between 0.4 and 0.5. The denominator is the number of dogs, so we want it to be as small a whole number as possible. The numerator is the number of female dogs. Trying fractions with a value less than one-half, no number of halves, thirds, fourths, fifths, or sixths has a value less than 0.5 but larger than 0.4. The first success is the fraction $3/7 \approx 0.4286$, so I own $\boxed{7}$ dogs.

Problem 4-3

I chose $\{50, 51, 52, \ldots, 97, 98, 99\}$. Here's why: Among the whole numbers from 1 through 99, there are 49 pairs whose sum is 100 (such as 1 and 99, 2 and 98, . . . , 49 and 51), so I chose at most one number from each such pair. For a maximum sum, I chose the larger number in each pair. Those are the largest 49 numbers. I also chose 50, since it had no pairmate with whom its sum was 100. I couldn't introduce another number without forming a pair whose sum was 100, so my selection's smallest number was $\boxed{50}$.

Problem 4-4



The shorter sides of each of the two small congruent right triangles are congruent. Each is labeled with an x. By the Pythagorean Theorem, $8^2+x^2 = (10-x)^2$. Solving, $x = 1.8$. Therefore, the length of the dotted line is $10-x = \boxed{8.2}$.

Problem 4-5

The number 2003 is a prime, so if $2003p+16 = x^2$, we can write $2003p = (x+4)(x-4)$. If $x+4 = 2003$, then $p = 1995$, which is not prime. If $x-4 = 2003$, then $p = x+4 = \boxed{2011}$.

[NOTE: None of $x+4 = \pm 1$, $x-4 = -2003$, or $x-4 = \pm 1$ yields a viable solution, but $x+4 = -2003$ also gives us $p = 2011$.]

Problem 4-6

Since $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = 1$, the factors $\sqrt{3}+\sqrt{2}$ and $\sqrt{3}-\sqrt{2}$ are reciprocals. If $r = \sqrt{3}+\sqrt{2}$, we will then have $r^2 = 5+2\sqrt{6}$, or $r^2-5 = 2\sqrt{6}$. Squaring, $r^4-10r^2+25 = 24$, or $r^4-10r^2+1 = 0$. Since $r = \sqrt{3}+\sqrt{2}$, it follows that $\frac{1}{r} = \frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{3}-\sqrt{2}$. Dividing $-r^4+10r^2 = 1$ by r, term by term, to get the $\frac{1}{r}$ we want, $-r^3+10r = \frac{1}{r}$, so $f(x) = \boxed{-x^3+10x}$.

[NOTE: It can be established, by consideration of each case separately, that no linear and no quadratic polynomial works. It can also be proven that the cubic polynomial we found above is unique.]

Contest # 5 *Answers & Solutions* 2/22/11

Problem 5-1

An equilateral $\triangle$ of side-length s has area $\frac{s^2\sqrt{3}}{4} = ks^2$. If two triangles have sides 3 and 4, then the sum of their areas is $k(3^2+4^2) = k(5^2)$. That's the area of an equilateral triangle of side-length $\boxed{5}$.

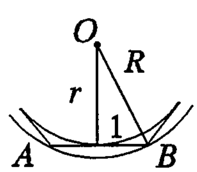
Problem 5-2

Set the numerator equal to the denominator to get $\frac{1}{m} - \frac{1}{n} = 1 - \frac{1}{mn}$. Multiplying by mn, $n-m = mn-1$. Rearranging, $(n+1) = mn+m = m(n+1)$. Thus, $(n+1)-m(n+1) = (n+1)(1-m) = 0$. Either $n = -1$ (not a positive integer) or $m = \boxed{1}$.

Problem 5-3

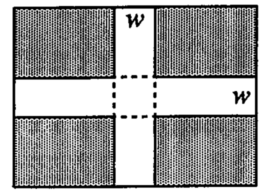
Let's examine what each girl said to see if it's consistent with only one girl lying. If B lies, then B is last. Then C is lying; *so B tells the truth.* If C lies, then C is not last. Then, A, B, D tell the truth, and nobody is last! That cannot be, *so C tells the truth.* If D lies, then D is first or last. But then A or C also lied, which is no good. Thus, *D tells the truth.* The liar was A. The order of finish was $BDAC$ or $BADC$. In either case, the winner was $\boxed{\text{Barb}}$.

Problem 5-4



In a quick sketch, the inscribed and circumscribed circles are given respective radius-lengths r and R. By the Pythagorean Theorem, $R^2-r^2 = 1$. Mutiplying by π, $\pi R^2 - \pi r^2 = \boxed{\pi}$.

Problem 5-5



The rectangle's width is 6. Its length is 8. Call each path's width w. Since the rectangle's area is $6\times 8 = 48$, the combined area of the two paths is half that, 24. One path's length is 6 and its area is $6w$; the other's length is 8 and its area is $8w$. The paths overlap in a square of area w^2. Thus, $6w+8w-w^2 = 24 \Leftrightarrow w^2-14w+24 = (w-2)(w-12) = 0$, so $w = \boxed{2}$.

Problem 5-6

Recall that $-1 \le \sin x \le 1$ and $-1 \le \cos y \le 1$.

Method I: If $a = \sin x$ and $b = \cos y$, then $k = a+b-ab = a+b(1-a) \le a+(1-a) = 1$, so $k \le 1$. Furthermore, $a+b(1-a) \ge a-(1-a) = 2a-1 \ge -3$, so $k \ge -3$. Any suitable value of k must satisfy $\boxed{-3 \le k \le 1}$.

[**NOTE**: If both $\sin x$ and $\cos y = -1$, then $k = -3$; if both $\sin x$ and $\cos y = 1$, then $k = 1$. Every value of k between -3 and 1 will arise as $\sin x$ and $\cos y$ take on their other values between -1 and 1.]

Method II: As above, $-1 \le a = \sin x \le 1$ and $-1 \le b = \cos y \le 1$. Since $a+b-ab = 1-(1-a)(1-b)$, and since $a \le 1$ and $b \le 1$, we know that $(1-a)(1-b) \ge 0$. Therefore, $1-(1-a)(1-b) \le 1$. Since $(1-a)(1-b) \le (1+1)(1+1) = 4$, we conclude that $1-(1-a)(1-b) \ge 1-4 = -3$. Finally, $1-(1-a)(1-b)$ takes on all values between -3 and 1.

Problem 6-1

The least common multiple of 1, 2, 3, 4, 5 is the product $2^2\times3\times5 = 60$. If I increase the list to include a 6 as well, the least common multiple stays the same since the factors in 6 already appear in $\boxed{60}$.

Problem 6-2

The given equation is valid for all x, including $x = 0$. If $x = 0$, $-92 = (-1)(-2)(-r)$, so $r = \boxed{46}$.

Problem 6-3

The length of any side of a triangle is less than the sum of the lengths of the other two sides. Therefore, $\log x < \log 4 + \log 503 = \log 2012$, or $x < 2012$. The greatest possible value of the integer x is $\boxed{2011}$.

[**NOTE**: The triangle inequality cited above also tells us that $\log 4 < \log 503x$ and that $\log 503 < \log 4x$, neither of which gives rise to an upper bound for x.]

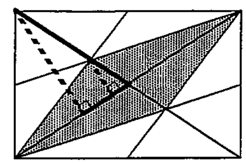
Problem 6-4

Since the three different uniform numbers add up to 21 and are equally spaced apart from each other, Bo's number, the middle number, is 7. Let the three uniform numbers be $7-x$, 7, and $7+x$. Al's number is $7-x$. Bo's new number is 6. Cy's new number is $8+x$. We're told that $\frac{8+x}{6} = \frac{6}{7-x}$, so $56+7x-8x-x^2 = 36$, or $x^2+x-20 = (x+5)(x-4) = 0$, so $x = -5$ or $x = 4$. Al's number is $7-x$. Since Al had the smallest number, his number was $7-4 = \boxed{3}$.

Problem 6-5

We're told that $b^2+g^2 = 697$. To get perfect squares whose sum ends in 7, we need squares that end in 1 and 6. Trial and error gives us the solutions $(b,g) = (11,24), (16,21)$. The class sizes would be $11+24 = 35$ and $16+21 = 37$. Together, the number of kids in the two classes is $35+37 = \boxed{72}$.

Problem 6-6



The diagonal shown splits the rectangle into two triangles, each having three medians drawn in its interior. The point at which all three medians are concurrent is 1/3 of the way from the midpoint to the vertex. Drop altitudes to the diagonal from the vertex and the point where the medians meet. By similar triangles, the altitude of each shaded triangle is 1/3 of the length of the altitude of the original triangle, so the shaded region's area = 1/3 of the area of the rectangle = $(1/3)(360) = \boxed{120}$.

[**NOTE:** Use the diagram below to show that, in the original rectangle, three adjoining unshaded regions can be rearranged to form one "unshaded" parallelogram. Each rectangle of area 360 can be rearranged into three congruent parallelograms, each of area 120.]